Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Discrete Mathematics

Subject Code: 4SC05DMC1 **Branch: B.Sc.(Mathematics)**

Semester: 5 **Date: 30/03/2017** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: **(14)**

a) Define: Modular lattice.

b) Draw a Hasse Diagram of $\langle S_6, \leq \rangle$ where $\leq$ - usual less than or equal to. Find the least and greatest element of it.

c) Prove that if $a = b$ then $ab' + a'b = 0$.

d) Define: Set of atoms and find $A(10)$ for Boolean algebra $\langle S_{30}, D \rangle$.

e) If $\alpha(x_1, x_2) = (x_1 \oplus x_2)'$ then find $\alpha(3,7)$ for $\langle S_{105}, D \rangle$.

f) Prove that in usual notation $\left(\underset{\sim}{A}'\right)' = A$.

g) Define: Difference of two fuzzy sets.

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions

a) i) Let $\langle L, \leq \rangle$ be a lattice $a, b \in L$ then prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$ **(07)**

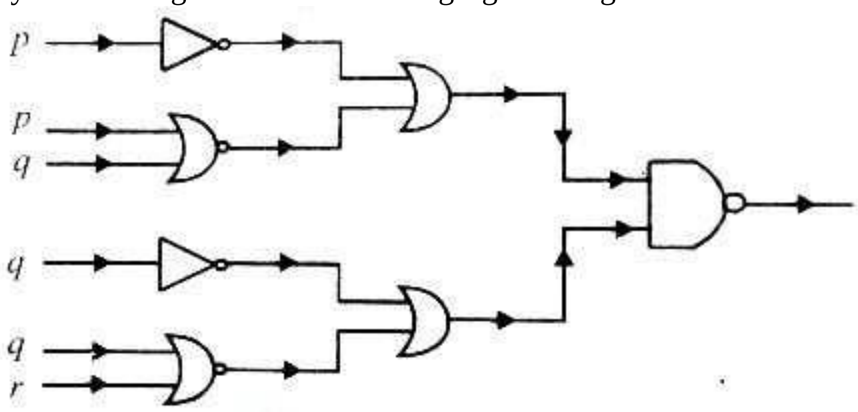
ii) Prove that Every distributive lattice is modular.

b) Define: Lattice as an algebraic system and If $\langle L, *, \oplus \rangle$ is a lattice an algebraic system then there exists an order relation $\leq$ on L such that $\langle L, \leq \rangle$ is a lattice as a poset. Where $a * b = \text{glb}\{a,b\}$, $a \oplus b = \text{lub}\{a,b\}$ for $\forall a, b \in L$. **(07)**

Q-3 Attempt all questions

a) Prove that $\langle S_{30}, GCD, LCM \rangle \cong \langle P(X), \cap, \cup \rangle$, where $X = \{a,b,c\}$. **(07)**

b) Simplify the circuit given in the following figure using Boolean identities: **(07)**



Q-4 Attempt all questions

a) For a lattice $\langle P(\{a,b,c\}),\subseteq\rangle$, answer the following questions: **(07)**

i) Find cover of each element and draw the Hasse diagram.
ii) Find lower bound, upper bound, greatest lower bound, least upper bound of $A=\{\{a\},\{a,b\}\}$.
iii) Find the least and greatest element of it.

b) Let $E=\{a,b,c,d,e\}$, $\underset{\sim}{A}=\{(a,0.3),(b,0.8),(c,0.5),(d,0.1),(e,0.9)\}$ and $\underset{\sim}{B}=\{(a,0.7),(b,0.6),(c,0.4),(d,0.2),(e,0.1)\}$ then find the following: **(07)**

1) $\underset{\sim}{A}\cup\underset{\sim}{B}$ 2) $\underset{\sim}{A}\cdot\underset{\sim}{B}$ 3) $\underset{\sim}{A}\hat{+}\underset{\sim}{B}$ 4) $\underset{\sim}{A}-\underset{\sim}{B}$ 5) $\underset{\sim}{A}\cap\underset{\sim}{B}$ 6) $\left(\underset{\sim}{A}'\right)'$ 7) $\underset{\sim}{B}'$

Q-5 Attempt all questions **(14)**

a) Obtain the product of sum canonical form of the following expressions in three variables by binary valuation tables $(x_1\oplus x_2)'\oplus\left(x_1'*x_3\right)$. **(05)**

b) Prove that $(a*b)'=a'\oplus b'$. **(05)**

c) Obtain the sum of product canonical form of the Boolean expression in three variables $\alpha(x,y,z)=x\oplus(y*z')$. **(04)**

Q-6 Attempt all questions

a) Let $\langle L,*,\oplus,0,1\rangle$ be a lattice and $a,b,c\in L$ then the $a\oplus(b*c)=(a\oplus b)*(a\oplus c)\Leftrightarrow a*(b\oplus c)=(a*b)\oplus(a*c)$. **(05)**

b) State D'Morgans laws for fuzzy subsets and prove any one. **(05)**

c) Find the cover of each element and draw Hasse diagram of $\langle L^3,\leq\rangle$; where $L=\{0,1\}$. **(04)**

Q-7 Attempt all questions

a) Prove that $\langle P(X),\subseteq\rangle$ is a lattice, Where $X=\{a,b,c\}$. **(05)**

b) Find the minimal sum of products expression for the function $f(x, y, z) = ab'c' + abc' + abc + ab'c + a'b'c$ by using Karnaugh map method. **(05)**

c) Obtain circuit diagram representation for the Boolean expression $\alpha(x, y, z) = y' + \left[z' + x + (yz)'\right](z + x'y)$. **(04)**

Q-8 State and prove Stone's representation theorem. **(14)**